Identification of rational function in s-domain describing a magnetic material frequency characteristics

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Abstract

High frequency magnetic cores are being used in different applications, from low-power electronic devices, up to high power components. For different material types it is required to identify material-specific equivalent circuit in order to simulate its behaviour in wide frequency range. This paper presents a method for obtaining such an equivalent circuit with the use of a rational function in s-domain such as it represents the material frequency characteristics. For the method presentation the measured data obtained for a real magnetic core was used.

1 Introduction

In order to analyse electrical circuits comprising magnetic cores (e.g. in transient power system studies) appropriate models of magnetic components are required. The models should reflect the core behaviour over wide frequency range. State-of-the-art methods on system identification are mostly based on Vector Fitting (VF) [1]. The Padé-based Approximation (PBA) method, first introduced by the authors in this paper, can be an interesting alternative to VF. In VF, the fitting process applies only to impedance-like rational functions, i.e. when degree of numerator n differs from degree of denominator m in a maximum of 1: $|n-m| \leq 1$. In PBA method, the fitting process applies not *only* to the impedance-like functions, but more in general, to *any* transfer function represented by a rational function with $n \leq m+1$.

Due to the use of Padé approximation, natural rational representation is the method output, providing direct input to Foster method of Lumped Element Equivalent Circuit (LEEC) synthesis [2] .

2 Frequency characteristics measurement

The method was tested for three different types of magnetic cores. Results for one of the cores are included in this paper. The input for the method are frequency characteristics measured with the use of frequency analyser in a range from 10 kHz to 100 MHz (see Figure 1).

The experimental characteristics $Z^{\exp}(f)$ consists of the amplitude $\mathrm{abs}(f)$ and the phase angle $\arg(f)$, such that $Z^{\exp}(f) = \mathrm{abs}(f) e^{j \arg(f)}$ (see Figure 2a-b, dots).

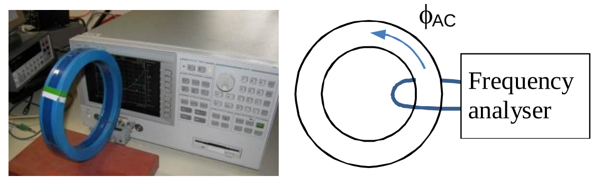

Figure 1 Experimental frequency characteristics; measurement set-up: magnetic core and impedance analyser

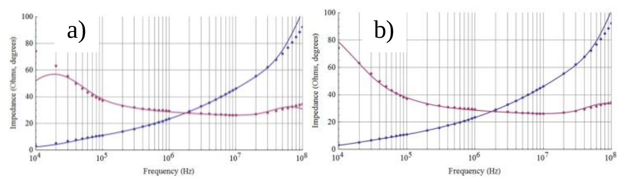

Figure 2 Frequency characteristics a magnetic core: measured (dots), approximated (lines)

3 Padé-based Approximation (PBA) method

3.1 Method outline

For LEEC synthesis the Foster method is used, where a rational function of the complex frequency $s \equiv j\omega$:

$$Z(s) = \frac{c_n s^n + c_{n-1} s^{n-1} + \cdots + c_1 s + c_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0} \qquad (1)$$

is expressed as a partial fraction decomposition and then directly transformed into LEEC, both structure and parameters.

In order to find a rational function (1), in PBA method the core is to find an auxiliary function $Z^{\mathrm{aux}}(s)$ *in s-domain* which fits to the experimental function $Z^{\exp}(f)$ obtained *in frequency domain*:

$$Z^{\exp}(f) \longrightarrow Z^{\mathrm{aux}}(s) \qquad (2)$$

In order to find the auxiliary function $Z^{\mathrm{aux}}(s)$ in s-domain, the challenge is that $Z^{\mathrm{aux}}(s)$ cannot be *any* function in complex domain, but it has to be built in a way that ω always stands beside j, so that $s \equiv j\omega$ is satisfied (i.e. Z^{aux} can be expressed by $Z^{\mathrm{aux}}(s)$).

Having the auxiliary function $Z^{\mathrm{aux}}(s)$, Padé approximation is used in order to fit rational function $Z(s)$ *in real domain* to $Z^{\mathrm{aux}}(s)$ in *s-domain*:

$$Z^{\text{aux}}(s) \xrightarrow{\text{Pade}} Z(s) \qquad (3)$$

For that purpose it is assumed that s is real and thus Padé approximation gives real $\{c_k\}$ and $\{b_k\}$ coefficients in (1) (otherwise the coefficients would be complex, and in such case $s \equiv j\omega$ would not be satisfied).

Having the rational function $Z(s)$, partial fraction decomposition is used in order to find equivalent circuit: both structure and parameters (Foster method).

3.2 Auxiliary function

For $Z^{\text{aux}}(s)$ the following function series was selected:

$$Z^{\text{aux}}(s) = \sum_{k=1}^{N} a_k\, \varphi_k(s) \qquad (4)$$

Conversion to s-domain representation requires that imaginary unit j cannot freely exist without ω (to make sure that $s \equiv j\omega$ is satisfied). This implies that coefficients of series representation $\{a_k\}$ must be real. $\{a_k\}$ also must be selected in a way so that the experimental data are at the same time approximated by both its real and imaginary parts:

$$\sum_{k=1}^{N} a_k\, \text{Re}(\, \varphi_k(j\, 2\pi f)) \cong \text{Re}(Z^{\text{exp}}(f)), \qquad (5)$$

and same for imaginary part.

For every k, functions $\text{Re}(\, \varphi_k(j\, 2\pi f))$ and $\text{Im}(\, \varphi_k(j\, 2\pi f))$ are real functions of frequency f and thus a standard approximation method in real domain can be used, namely Least Squares Function Approximation. The method gives real $\{a_k\}$ coefficients, for which at the same time both conditions (5) are met. These are obtained by solving the following equation (scalar product of $\varphi_k(s)$ real and imaginary parts):

$$\begin{pmatrix} s_{1,1}^{\text{Re}}+s_{1,1}^{\text{Im}} & s_{1,2}^{\text{Re}}+s_{1,2}^{\text{Im}} & \cdots & s_{1,N}^{\text{Re}}+s_{1,N}^{\text{Im}} \\ s_{2,1}^{\text{Re}}+s_{2,1}^{\text{Im}} & s_{2,2}^{\text{Re}}+s_{2,2}^{\text{Im}} & & s_{2,N}^{\text{Re}}+s_{2,N}^{\text{Im}} \\ \vdots & & \ddots & \vdots \\ s_{N,1}^{\text{Re}}+s_{N,1}^{\text{Re}} & s_{N,2}^{\text{Re}}+s_{N,2}^{\text{Im}} & \cdots & s_{N,N}^{\text{Re}}+s_{N,N}^{\text{Im}} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} t_1^{\text{Re}}+t_1^{\text{Im}} \\ t_2^{\text{Re}}+t_2^{\text{Im}} \\ \vdots \\ t_N^{\text{Re}}+t_N^{\text{Im}} \end{pmatrix} \quad (6)$$

3 Method verification

In Figure 2 the results of approximation are shown (lines) for a core presented in Figure 1. If (n, m) stands for a rational function such as the degree of numerator (denominator) is $n(m)$, then Figure 2a presents results for rational functions of type (7,7) and Figure 2b – for (10,10). The Mean Relative Error (MRE) for (7,7) is (3.79%, 3.42%) for amplitude and phase respectively. Same for (10,10) is (1.86%, 1.67%).

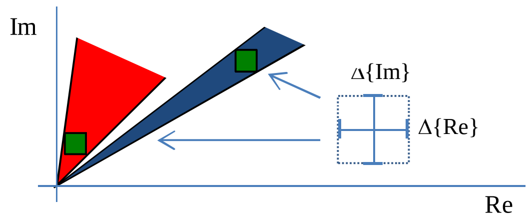

Figure 3 Dependence of phase angle error with amplitude

Figure 3 gives explanation for the worse phase fit achieved in low amplitudes range.

4 Lumped element equivalent circuit

Based on the rational function a lumped elements equivalent circuit was composed with lumped passive elements (resistances and inductances). Foster method was used [2], where the rational function in s-domain (as obtained in the previous sections) was decomposed into partial fractions so that each partial fraction was directly modelled by parallel connected L and R elements (all L-R two-terminal circuits are connected in series):

$$Z(s) = R_0 + \sum_{i=1}^{7} \frac{R_i \cdot s}{R_i / L_i + s}. \qquad (7)$$

Figure 4 presents the equivalent circuit for approximation (7,7) implemented in EMTP-ATP software. Figure 5 presents frequency characteristics calculated for the circuit in Figure 4 and shows good agreement with measured data in Figure 2.

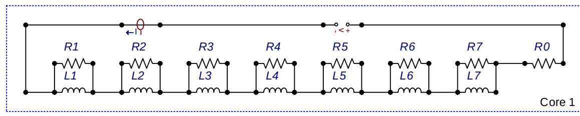

Figure 4 Lumped element equivalent circuit for core 1 (7,7)

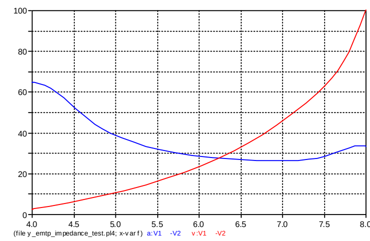

Figure 5 Frequency characteristics calculated in EMTP-ATP

5 Conclusions

New method of Padé-based Approximation (PBA) was presented with its application for lumped element equivalent circuit (LEEC) modelling. Advantages of the method include wider scope of its applications than for the well-established method of Vector Fitting (VF).

The method was applied and tested in modelling of frequency characteristics of a nanocrystalline cores based on the measurements presented also in the paper.

References

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